1. **Discrete to Continuous**

Continuous random variables can take any value in a range

The probability that a continuous random variable takes a specific value is zero

Its probabilities are determined by a pdf, which is non-negative and the area under the curve is equal to 1

The probability that it lies between c and d is the area under the pdf between c and d

1. **Elicitation – the process of getting or producing something, especially information or a reaction**

Bayesians express uncertainty through probability distributions

One can self-elicit a probability distribution that reflects your personal probability

Personal probability should change as new data are observed

The beta family of distributions can flexibly express many possible beliefs about p

1. **Conjugacy (conjugate (v) – paired, or equally coupled, working in union)**

Conjugacy occurs when your new belief that is the posterior distribution is in the same family as your prior belief but with new parameter values

Eg. Prior belief on the probability of winning in a game of pocker is Beta(3,2). You play five games and win four => the new belief is Beta(3+4, 2+5-4)

* 1. **Binominal proportion inference**

Eg. Frequentist inference: tossing a coin 4 times and each time it landed with a head side.

H0: p>= 0.5 Ha: p<0.5

If H0 is true then p value = 0.54 = 0.0625 > 0.05 => do not reject null hypothesis

Eg. Bayesian inference: Prior belief on the probability of winning in a game of pocker is Beta(3,2). You play five games and win four => the new belief is Beta(3+4, 2+5-4). The mean of this Beta distribution is 7/(7+3).

* 1. **Gamma-Poisson conjugate families**

Parameters for the pdf for the Poisson is lambda

Gamma distribution describe continuous non-negative random variables

Parameters for the pdf for the Gamma is k and theta. Mean = k\*theta, std.dev. = theta\*

New parameters for Gamma-Poisson conjugate: Observations x1, x2, …, xN

K\* = k +

Assume the number of Academy Award winners who die in a month has a Poisson distribution with mean lambda. You don't know lambda, but your prior is the gamma distribution with parameters k = 0.25 and theta = 6. Next month 2 Academy Award winners die. What is your new posterior mean?

Answer: k\*=0.25+2=2.25 = 6/(1\*6+1)=6/7

* 1. **Normal-normal conjugate families**

Assumption:

-Prior on unknown mu is normal

Mean v(nu)

Standard deviation t (tau)

-Data x1, x2, …, xN are independent

Come from normal with standard deviation, sigma

**CREDIBLE INTERVALS AND PREDICTIVE INFERENCE**

1. **Non-conjugate priors**

Not everything is conjugate

Computation allows Bayesian inference when personal probability and data distribution are not conjugate (JAGS)

1. **Credible intervals**

Confidence intervals: 95% ci on the mean means “95% of similarly constructed intervals will contain the true mean” NOT “the probability that the true mean lies between L and U is 0.95” – for frequentist

“The probability the true mean is contained within a given interval is 0.95” – for Bayesian

1. **Predictive inference**

**Question:**  What is the probability that a fifth child born in the RU-486 trial will have a mother who received RU-486?

What is the probability that your stock broker’s next recommendation will be profitable?

Example: you have 2 coins, p1(head)=0.7, p2(head)=0.3.

Prior belief: probability of having the first coin is 50%.

Take random 1 coin from a bag and toss 2 times, both are head. What is the posterior belief about the probability that you took the first coin?

P\* = P[2 heads|0.7] \* 0.5/ (P[2 heads|0.7] \* 0.5 + P[2 heads|0.4] \* 0.5) = 0.754

* Probability of taking the second coin from the bag is 1 – 0.754 = 0.246
* Predictive: Probability of having head when toss a coin = P[heads|0.7]\*0.754 + P[heads|0.4] \* 0.246